PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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A Magnetic Detector of Electrical Waves and Some of Its Applications

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PHILOSOPHICAL TRANSACTIONS.

I. *A Magnetic Detector of Electrical Waves and some of its Applications.*

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INTRODUCTION.

THE present paper deals with the subject of the magnetization of iron by high-frequency discharges, and the uses of magnetized steel needles for detecting and measuring currents of very great rapidity of alternation.

It will be shown that these magnetic detectors offer a very simple means of investigating many of the phenomena connected with high-frequency discharges, and may be used over a wide range of periods of alternation. Not only may these detectors be used in ordinary Leyden jar circuits, but they also offer a sensitive means of investigating waves along wires and free vibrating circuits of short wave-lengths.

They were also found to be a sensitive means of detecting electrical radiation from Hertzian vibrators at long distances from the vibrator.

In the course of the paper the following subjects are investigated :—

I. Magnetization of iron by high-frequency discharges and the investigation of the effect on short steel needles.

II. Magnetic detectors and their uses.

a. Detection of electro-magnetic radiation in free space.
  Waves were detected over half-a-mile from the vibrator.

b. Waves along wires.

c. Damping of oscillations.
  Resistance of iron wires.
  Absorption of energy by conductors.

d. Determination of the period of Leyden jar discharges and the constants of the discharge circuit.

The magnetization of steel needles when placed in a spiral through which a Leyden jar discharge was passed has long been known.

In 1842 Professor HENRY was led to suspect from the anomalous magnetization of steel needles that the Leyden jar discharge was oscillatory.

Professor HENRY, ABRIA, and several others, used steel needles in their attempts to determine the direction of induced currents in secondary and tertiary circuits, when the Leyden jar was discharged through the primary, but with conflicting results.

Lord RAYLEIGH ('Phil. Mag.,' vol. 39, 1870, p. 429) made use of steel needles in a magnetizing spiral in investigating the maximum current of a break for ordinary induction circuits.

The general subject of the magnetization of iron, for rapid oscillatory currents, has been worked at by many different experimenters; Lord RAYLEIGH, using oscillatory currents of a frequency up to 1050 a second, showed that iron wires showed considerable increase of resistance, and deduced the value of the permeability of the wire. TROWBRIDGE ('Phil. Mag.,' 1891) has shown that iron wires rapidly damp down the oscillations of the Leyden jar discharge, and from his results deduced a rough value for the permeability of the specimens tested.

V. BJERKNES ('Electrician,' November 18, 1892) found that the damping out of oscillations in a Hertzian resonator takes place much more rapidly in a resonator of iron than when it is made of non-magnetic material.

Magnetization of Iron by a Leyden Jar discharge.

If a piece of steel wire, several centimetres in length, be taken and placed in a solenoid of a few turns, on the passage of a discharge the wire will be found to be magnetized. The magnetization is, in general, small, and increases slightly in amount when a succession of discharges are passed in the same direction.

Fig 1.

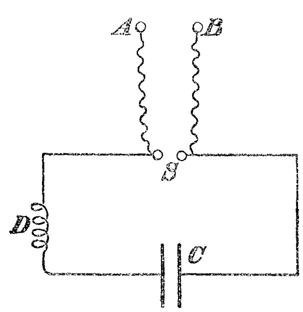


Fig. 1 shows the arrangement. A and B are the poles of a Wimshurst machine or an induction coil, C the condenser, S the air-break, and D the solenoid in which the steel wire is placed. The charging-up of the condenser before the spark passes was found to have no effect in magnetizing the needle.

In all experiments to follow, the magnetization of the needles was tested by means of a small mirror magnetometer. The needle was either fixed in position in the solenoid and the magnetometer placed beside it, or the needle removed and tested after each experiment. If wires of the same length but of different diameters be taken, it will be found that the magnetization is roughly proportional to the diameter of the wires. This is to be expected, since the magnetizing forces are confined to a thin skin on the surface of the needle, and so the amount of magnetization depends more on the surface than on the sectional area.

In order to determine accurately the way in which a piece of steel was magnetized from the surface inwards, recourse was had to a method of solution of the iron in acid. The needle to be tested was fixed in a glass vessel before a dead-beat magnetometer. Dilute hot nitric acid was poured in and kept at a constant temperature. As soon as the needle was covered it commenced to dissolve, and the variation of the deflection with the time was noted. In this way the amount and stages of the magnetization of the iron could be completely determined. From preliminary experiments on *uniformly* magnetized needles, it was found that under the action of the acid the diameter of the wire decreased uniformly with the time.

Let I represent the intensity of magnetization of a thin circular shell distant r from the centre of the needle, and M the deflection of the magnetometer at any instant.

$$\int_0^r \mathrm{I} \, . \, 2\pi r \, dr \text{ is proportional to M,}$$

therefore

$$\mathrm{I}r \text{ is proportional to } d\mathrm{M}/dr.$$

Let r be the radius of the wire at first. It has been shown that $a - r$ is proportional to t, the time of action of the acid.

Therefore

$$a - r = \kappa t \text{ where } \kappa \text{ is a constant,}$$

and

$$- dr = \kappa dt,$$

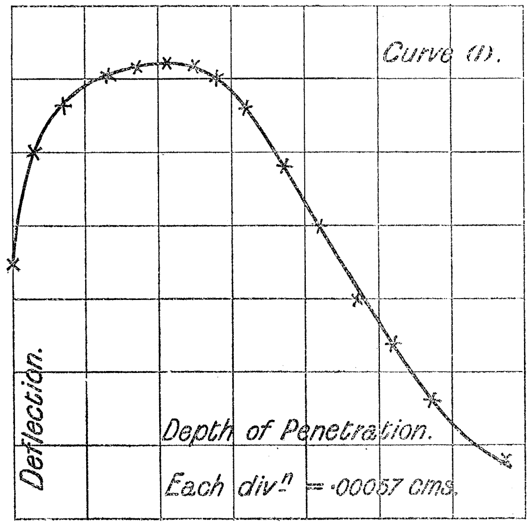
therefore

$$\mathrm{I} \text{ varies } \frac{1}{a - \kappa t} \, . \, \frac{d\mathrm{M}}{dt}.$$

If a curve be plotted whose ordinates represent the deflection and the abscissæ the time of action of the acid, $d\mathrm{M}/dt$ at any point is equal to the tangent of the angle which the tangent to the curve at that point makes with the axis of x. The variation of I can thus be completely determined from the experimental curve.

The following curve (Curve I) is an example of the magnetization of a piece of pianoforte wire, 4 centims. long, ·08 centim. in diameter, placed in a solenoid of two turns per centim. The frequency of the discharge was about 3 million per second.

The ordinates of the curve represent the deflection of the magnetometer and the abscissæ the depth to which the iron has been dissolved by the acid, measuring from the surface inwards. Each division of the ordinates corresponds to a depth = ·00057 centim.



The deflection of the magnetometer at first was 85 divisions. As the needle commenced to dissolve, the deflection increased rapidly to 156, remained nearly steady for a short time, and then rapidly diminished to zero; when this was the case the diameter of the needle was ·032 centim., so that the magnetization had penetrated to a distance of ·024 centim.

If the variation of the value of I, the intensity of magnetization from the surface inwards, be deduced from this curve, it will be seen that the surface layer is magnetized in one direction and an interior layer in the opposite direction. This apparently gives evidence of only two half oscillations, in opposite directions, in the discharge. A large number of needles were dissolved after magnetization under various conditions, and the same peculiarity was observed, although, from other evidence, it was known that there were a large number of vigorous oscillations before the discharge was much damped down.

When a needle magnetized to saturation was subjected to the discharge, the magnetization of the needle was always diminished, and on solution of the iron the same effect was observed, viz., a surface layer magnetized in the opposite direction to the internal magnetism.

Since a Leyden jar discharge in general gives several complete oscillations before it is greatly damped down, it would be expected that the surface layer of a uniformly magnetized needle would either be completely demagnetized or show evidence of several oscillations in opposite directions. The effect observed may be explained when the demagnetizing force of the ends of a short needle on itself is taken into account. The first half oscillation that tends to demagnetize the needle has the demagnetizing force of the needle assisting it, while the return oscillation has it in opposition. The return oscillation will therefore not be able to completely remagnetize the surface layer already affected, but a thin layer will be left in the interior. After the passage of the next oscillation another layer will be added in the same direction, and so on, till the final effect will be that the surface of the needle will be magnetized in the opposite direction to the interior.

If strongly-magnetized needles of the same diameter, but of different lengths, are taken and placed in the same solenoid, it will be found that the reduction of magnetic moment of the needle, due to the discharge, is greater the shorter the needle. This effect is due to the demagnetizing influence of the ends, which is greater the shorter the needle.

It was also found that if successive discharges be passed, the reduction of deflection gradually increases, till it reaches a steady state, so that the passage of any further number of discharges has no apparent effect on the magnetism of the needle.

The following table shows the effect of varying the length of the needle, the diameter being kept constant, and also the effect of successive discharges in each case.

Needle ·08 centim. in diameter; frequency about 3 millions.

Number of discharges.	10·5 centims.	6·4 centims.	3·2 centims.	1·6 centims.	·75 centim.
0	250	250	250	250	250
1	204	190	166	150	114
2	199	182	155	135	88
5	195	175	138	115	64
10	190	170	130	107	57
20	189	166	125	102	54
50	188	162	120	98	50

In the above table each of the needles was placed at such a distance from the magnetometer to give the same steady deflection of 250. The vertical columns show how the deflection fell after the passage of the different numbers of discharges. The vertical columns correspond to needles 10·5, 6·4, 3·2, 1·6, ·75 centim. respectively. For the needle 10·5 centims. long, the deflection fell from 250 to 188, while for the short needle, ·75 centim. long, the deflection fell from 250 to 50, although all the other conditions were precisely the same for each.

It will be observed from the above that the first discharge is mainly instrumental in reducing the deflection, and that after ten discharges have been passed, the deflection has nearly reached its final value.

Whenever a *magnetized* needle is placed in a solenoid and a discharge passed, there is always a reduction of the magnetization, the amount depending, for any given size of needle, on the intensity of the magnetic force in the solenoid and on the period of the oscillation. This is the case whether we are dealing with Leyden jar circuits, or the free vibrations, such as are set up in Hertzian receivers.

An unmagnetized needle, on the other hand, is not appreciably magnetized when placed in a circuit where the damping is small, for each successive oscillation destroys the effect of the previous one. Soft iron wires exhibit a similar effect to steel, only it is difficult to use wires of sufficient length to retain their magnetization. The effect on the needle is in general a purely surface one, and the amount of demagnetization does not bear any simple relation to the magnetizing force acting on it.

After every experiment, the needle was removed and placed in a solenoid and a steady current passed, sufficient to saturate the steel. In this way the needle could always be quickly reduced to the same state after any experiment, and it was found that, using hard steel wires, the results of successive experiments were very consistent with one another.

Passage of a Discharge Longitudinally through a Magnetized Wire.

If a piece of pianoforte wire, several centimetres in length, be taken and placed in series with the discharge circuit, in the passage of a discharge, the magnetic moment of the needle is diminished, due to the "circular" magnetization of the wire. If the needle be dissolved in acid, it will be found that there is a thin skin, apparently magnetized in opposition to the original magnetization, due to the resultant action of the demagnetizing force of the needle and the magnetic force due to the current in the wire.

The magnetic force H acting at any given point in the wire is given by

$$H = \frac{2\gamma}{r},$$

where γ is the current through the conductor flowing internal to the circle described through the point, and concentric with the surface of the conductor. The value at the surface of the wire is given by

$$H = \frac{2\gamma}{a},$$

where a is the radius of the wire.

Assuming μ, the permeability of the iron wire, as constant, the maximum value of the current at any point of the conductor decreases in geometrical progression as the distance from the surface inwards increases in arithmetical progression. As will be shown later in the part on "Resistance of Iron Wires," the current falls off even more rapidly than the theoretical law, on account of the increase of the value of μ as the amplitude of the current diminishes in intensity from the surface inwards.

For thin wires the magnetic force at the surface of the wire is much greater than for thicker ones. We should, therefore, expect a thick magnetized wire conveying the current to be affected to less depth than a thin one, and this is found to be the case.

A thin steel wire, ·025 centim. in diameter, was completely demagnetized by a discharge. In this particular case the maximum value of the current through the wire was about 100 amperes, and the value of the magnetic force at the surface of the wire was, therefore, about 1600 C.G.S. units. A hard steel wire, ·08 centim. in diameter, was only partially demagnetized, the deflection being reduced from 250 to 116 scale divisions.

The following are examples of a few of the experiments on the demagnetization of iron wires when the frequency of the discharge was about 3 million and the value of the maximum current about 100 amperes :—

1. Thin soft iron, Thin steel } wire diameter, ·025 centim. : completely demagnetized.
2. Steel wire : diameter, ·08 centim. : fall of deflection from 250 to 116.
3. Steel wire : ,, ·16 centim. : ,, ,, 250 to 184.
4. Steel wire : ,, ·25 centim. : ,, ,, 250 to 216.
5. Long hollow soft iron cylinder, $\frac{1}{4}$ millim. thick and diameter 1·8 millims.: fall of deflection from 250 to 230.

The same condenser and discharging current were used for all the specimens tested, and it is of interest to observe the depth to which the magnetism of the iron was affected by the discharge, assuming that the final deflection is that due to the mass of iron not circularly magnetized.

Wire.	Diameter.	Depth of penetration of the discharge.
	centim.	centim.
Hard steel wire . . .	·08	·013
Soft steel wire	·16	·011
Soft steel	·25	·009
Soft iron cylinder . . .	1·8	·0011

Experiments of this kind show to what a small depth the current penetrates into the iron wire. Very large momentary currents are conveyed through a surface skin of the conductor, and the intensity of the current diminishes rapidly inwards.

A thin magnetized steel wire was placed in the circuit of a small Hertzian plate vibrator. The deflection due to the needle fell from 300 to 250 after a succession of discharges.

This shows that the iron was unaffected below a depth of about ·0011 centim.

For rough comparisons of the intensity of currents in multiple circuits, the use of the "longitudinal" detector is often preferable to placing the needle in a solenoid.

A thin magnetized steel wire, placed in series with the circuit, is a surprisingly sensitive detector of oscillatory currents of small intensity.

In practice copper wires were soldered on to the extremities of the steel needle, which is placed in position before a magnetometer. A magnetizing solenoid is wound over the needle, and after every experiment a steady current was sent through in order to re-saturate the needle.

Both the "longitudinal" and "solenoidal" detectors may be very readily used to compare the intensities of currents in multiple circuits when the period of oscillation is the same for each. The best form of the solenoidal detector is explained later, and it has the advantage of being able to distinguish between the intensity of the first and second half oscillations.

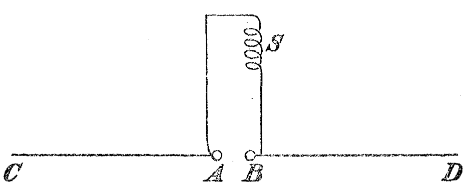
Detection of Waves in Free Space.

It has been shown that the amount of demagnetization of a magnetized needle depends on the fineness of the wire and the number of turns per centim. on the magnetizing solenoid.

If a short piece of thin magnetized steel wire be taken, and a large number of turns wound over it, it is a very sensitive means of detecting electrical oscillations in a conductor when the amplitude of the oscillations is extremely small. It was on this principle that a detector for electrical waves was devised, which proved to be a sensitive means of detecting Hertzian waves at considerable distances from the vibrator.

About twenty pieces of fine steel wire ·007 centim. in diameter, each about 1 centim. long, and insulated from each other by shellac varnish, formed the detector needle. A fine wire solenoid was wound directly over it, of two layers corresponding to about 80 turns per centim. As the solenoid was of very small diameter, about 15 centim. of wire served to wind the coil. This small detector was fixed at the end of a glass tube, which was itself fixed on to a wooden base, the terminals of the detector coil being brought out to mercury cups.

Fig. 2.



S (fig. 2) represents the detector needle and the solenoid wound over it. A and B are the mercury cups. CA and BD were two straight rods which served as receivers, one end of each being placed in the mercury cups.

The detector needle was strongly magnetized and placed before a small magnetometer, the deflection due to the needle being compensated by an auxiliary magnet.

If the receiving wires were parallel to the electric force of the wave from the vibrator, oscillations were set up in the receiver circuit, the surface layers of the needles were demagnetized, and there resulted a corresponding deflection of the magnetometer needle.

The amount of the deflection, of course, depended on the amplitude of the oscillations set up in the receiver, and, therefore, on the distance from the vibrator.

Long Distance Experiments.

When a Hertzian vibrator was used with plates 40 centims. square, and a short discharge circuit, quite a large deflection was obtained at a distance of 40 yards, the waves passing through several thick walls between the vibrator and receiver.

Further experiments were made to see how far from the vibrator electromagnetic radiation could be detected.

For the long distance experiments, the vibrator consisted of two zinc plates, 6 feet by 3 feet, and separated by a short discharge circuit of about 30 centims. When large plates were used, a Wimshurst machine was equally efficient as a Ruhmkorff coil for exciting the vibrations.

The first experiments were made over Jesus Common, Cambridge, the receiver being placed in one of the buildings on Park Parade. Quite a large effect was obtained at a distance of a quarter of a mile from the vibrator, and from the deflection obtained it was probable that an effect would have been got for several times that distance.

When the vibrator was set up in the top floor of the Cavendish Laboratory, a small, but quite marked effect was obtained at Park Parade, a distance of over half a mile in the direct line.

In this case the waves, before they reached the receiver, must have passed through several brick and stone walls, and many large blocks of buildings intervened between the vibrator and receiver. The length of wave given out by the vibrator was probably six or seven metres, and a wave of that length seemed to suffer very little loss of intensity in passing through ordinary brick walls.

From an experiment tried in the Cavendish Laboratory it was found that the effect of six solid walls and other obstacles between the vibrator and receiver did not diminish the effect appreciably. When the vibrator was working in the upper part of the Laboratory, a large effect could be obtained all over the building, notwithstanding the floors and walls intervening.

A large number of experiments were made on the effect of varying the length and diameter of the receiving wires. If a fairly dead-beat vibrator were used, *e.g.*, plates with a short inductance, it was found that the deflection gradually increased with increase of length of the receiving wires, reaching a maximum which was unchanged by any further increase of length.

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The effect on the detector was found to be practically independent of the sectional area of the receiving wires. A thin wire and a thick rod of the same length had equal effects; a plate of metal, 6 centims. wide, produced the same deflection as a thin wire.

If two wires instead of one were used in parallel the effect was the same as one, though the wires were some distance apart. Any number of wires in parallel had the same effect as a single wire or plate.

No difference could be detected whether the first half oscillation in the receiver tended to magnetize the needle or the reverse. Since the vibrator used was nearly dead-beat, this shows that the damping of the oscillations in the receiver is very small. On introducing a short carbon rod in the circuit the deflection was greatly reduced.

It was found impossible to magnetize soft iron or steel when placed in the receiving circuit on account of the slow decay of the amplitude of the oscillations. The detector needle may be kept in position for a succession of observations, provided the current in the receiving circuit is steadily increasing for each experiment, otherwise the detector should be remagnetized and placed in position again after each observation. The deflection was found to be very constant for a series of experiments under the same conditions.

The connection between the intensity of the electric force at the receiver and the deflection of the magnetometer needle can be easily determined by swinging the receiving wires through different angles.

When the receiver is placed symmetrically with regard to the vibrator, the deflection was a maximum when the receiving wires were parallel to the axis of the vibrator, and the intensity of the electric force acting along the receiver varies as the cosine of the angle from the maximum position.

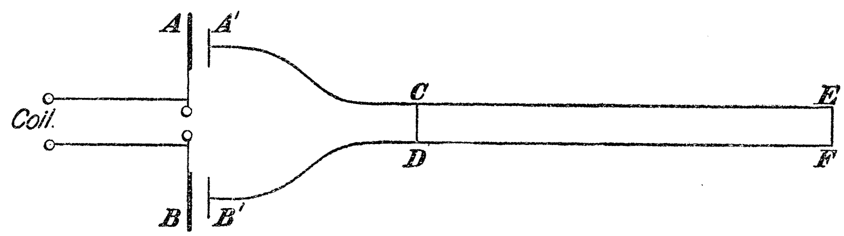
With plate vibrators the deflection was found to be nearly independent of the degree of brightness of the spark terminals and remained sensibly constant for long intervals. In the case of the small cylindrical vibrator used by HERTZ with the parabolic reflectors, the deflection continually varied with the state of the sparking terminals, and such small vibrators cannot be relied on for metrical experiments.

Some experiments were made to see if the magnetic force in the wave front could be directly detected. A collection of thin wires, insulated from each other and magnetized to saturation, were used and placed in the direction of the magnetic displacement, but the values of the magnetic force were too small to be observed, except quite close to the vibrator.

Waves along Wires.

It was found that the use of a detector, composed of fine insulated wires, was quite delicate enough to investigate waves along wires when there was only one turn of wire round the detector needle. Since the reduction of magnetic moment was nearly proportional to the amplitude of the current, the intensity of currents at various points could be very approximately compared.

Fig. 3.



The ordinary Hertz arrangement (fig. 3) was set up for obtaining free vibrating circuits.

A and B were two plates set up vertically. Beside them were two small plates A′ and B′, and long wires A′E, B′F were led from these plates. A fixed bridge was placed at EF, the ends of the wires, and a detector placed at the middle point of EF, with a small magnetometer fixed in position.

A sliding bridge, CD, was then moved till the fall of the deflection of the detector needle was a maximum. This position of the bridge could be very accurately determined, for a movement of the bridge through 1 centim. altered the deflection considerably. The detector was then placed in various parts of the circuit CE, and the amplitude of the current at the different points determined. It was found that the current was a maximum at C and the middle point of EF. A well-defined node was found at the middle point of CE.

The length CEFD was thus half a wave-length.

Since the use of the bolometer has been the only means of accurately investigating waves along wires, it was interesting to observe whether the magnetic detectors were of the same order of sensitiveness as the bolometer.

One turn of wire was wound round each of two glass tubes, sliding along the wires CE and DF, as in Reuben's experiments. Instead of the fine bolometer wire, a detector needle, with several turns of wire around it, was placed in series with the two turns of wire. The charging and discharging of the small condensers, formed by the straight wires and the small coils around them, was quite sufficient to almost completely demagnetize the needle. By this method the movement of extremely small quantities of electricity could be detected and the sensitiveness was quite comparable with that of a delicate bolometer.

No appreciable damping could be detected for the long wire circuits, showing that they were probably vibrating almost independently of the primary vibrator.

If a metre or two of wire was fixed to the pole of a Wimshurst machine, on the passage of a spark, there was evidence of a rapidly-oscillating current set up in the wire. By using a sensitive detector, with a few turns round it, the variation of the current along the wire could readily be determined.

If short lengths of wire were fixed to any portion of a Leyden jar circuit, on the passage of a discharge, there was always evidence of a rapid oscillation set up in the wire. Each of the short circuits had a tendency to vibrate in its own natural period, but the results were complicated by the oscillations of the main circuit.

Damping of Oscillations.

The use of magnetized needles offers a simple and ready means of determining the damping of oscillations in a discharge circuit.

Let L be self-inductance of discharge circuit for rapid currents.

" $C =$ capacity of condenser.

" $R =$ resistance of leads and airbreak to the discharge.

" $V_0 =$ potential to which condenser is charged.

The current γ at any instant is given by

$$\gamma = \frac{CV_0}{(LC)^{\frac{1}{2}}} e^{-R/2L \cdot t} \sin \frac{t}{(LC)} .$$

The exponential term only includes the case of frictional dissipation of energy, and does not take into account radiation into space. In the experiments at present considered, where the condenser is of the type of a Leyden jar, there can be but very small amount of dissipation of energy due to radiation.

Assuming R to be constant, the amplitude of the current decays in geometrical progression.

Consider two similar small oppositely wound solenoids A and B placed in series in the discharge circuit. Two magnetized needles are placed in A and B, the north poles facing in the same direction. After the passage of a discharge, it will be found that the reduction of magnetic moment is not the same in the two needles.

Let $\alpha_1 \alpha_2 \alpha_3 \ldots$ be the half-oscillations of the discharge in one direction.

" $\beta_1 \beta_2 \beta_3 \ldots$ be the half-oscillations in the opposite direction.

Suppose that the half-oscillation α_1 tends to magnetize the needle in the solenoid A still further. Since the needle is saturated no effect is produced. β_1 demagnetizes the surface skin, α_2 tends to remove the effect of β_2, and so on. In the solenoid B α_1 demagnetizes the needle, β_1 tends to remagnetize it in its original direction, and so on. Since the maximum value of the current of α_1 is greater than the maximum value of β_1, the needle in B will be more demagnetized than in A.

If, however, we increase the number of turns per centimetre on the solenoid A, until the effects on the two needles are exactly the same, then *assuming* that the value of the current decreases in geometrical progression, the maximum value of the magnetic force due to the oscillation β_1 acting on the needle A is equal to the maximum value due to the oscillation α_1 on β.

Let $\gamma_1\gamma_2$ be the maximum values of the current in the first and second half-oscillations respectively.

Let $n_1 n_2$ be the number of turns per centimetre on solenoids A and B respectively.

Then, since

$$4\pi n_1\gamma_1 = 4\pi n_2\gamma_2, \qquad \gamma_2/\gamma_1 = n_1/n_2,$$

the ratio of the second to the first half-oscillation is therefore known, and the damping is thus determined. The actual resistance in the circuit may also be deduced.

Now $\gamma_1 = pCV_0 e^{-R/2L\,.\,T/4}$ where T = period of complete oscillation and $p = \frac{1}{(LC)^{\frac{1}{2}}}$,

$\gamma_2 = pCV_0 e^{-R/2L\,.\,3T/4}$.

Therefore

$$e^{-R/2L\,.\,T/2} = \frac{\gamma_2}{\gamma_1} = \rho_1, \text{ say.}$$

Therefore

$$\log \rho_1 = -\frac{R}{2L}\cdot\frac{T}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Since L and T are known from the constants of the discharge circuit, and ρ_1 is determined by experiment, R is known.

In practice, in order to avoid the necessity of determining the constants of the discharge circuit, an additional known resistance r is introduced into the circuit. If an electrolytic resistance of zinc sulphate with zinc electrodes be used, the resistance will be found to be practically the same for steady as rapidly alternating currents, as the specific resistance is very great.

Let ρ_2 be the ratio of the amplitudes of the two half oscillations when $R + r$ is in the circuit

$$\log \rho_2 = -\frac{R + r}{2L}\cdot\frac{T}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Dividing (2) by (1)

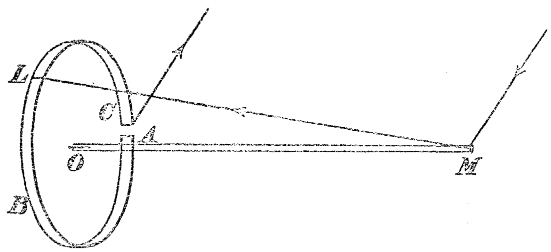
$$\frac{R + r}{R} = \frac{\log \rho_2}{\log \rho_1},$$

R is therefore determined in term of r, a known resistance.

The method of two solenoids was not adopted in practice, but one theoretically equivalent employed.

A narrow piece of sheet zinc ABC was taken (fig. 4) and bent into almost a complete circle of 7 centims. diameter. This was fixed on a block of ebonite. At the centre of the circle a thin glass tube OM was placed, which served as the axis of a metal arm LM, which pressed against the circumference of the circle and could be moved round it. The "detector" consisted of about thirty very fine steel wires, ·003 inch in diameter, arranged into a compound magnet about 1 centim. long. The wires were insulated from each other by shellac varnish, and the small needle was fixed inside a thin glass tube which could be easily slipped in and out of the central glass tube OM.

Fig. 4.



A divided scale was placed round the circumference ABC, and the whole arrangement was fixed in position before a small mirror magnetometer. The magnetized "detector" needle was placed in position by sliding it in the glass tube, and the deflection due to the needle was compensated by another magnet. The wires of the discharge circuit were connected to C and M, and when the arm ML was at C no effect was produced on the needle. When the discharge passed round the circle there was a deflection due to the partial demagnetization of the detector. The detector was then quickly removed and magnetized to saturation in an adjacent solenoid and then replaced. It was found that, provided the detector was magnetized in a very strong field, on replacing it in position the zero remained unchanged, and the same deflection was obtained time after time for similar discharges.

Since the magnetic field at the centre of a circle due to an arc of length l is given by

$$H = \frac{l\gamma}{r^2}, \text{ where } \gamma \text{ is the current,}$$

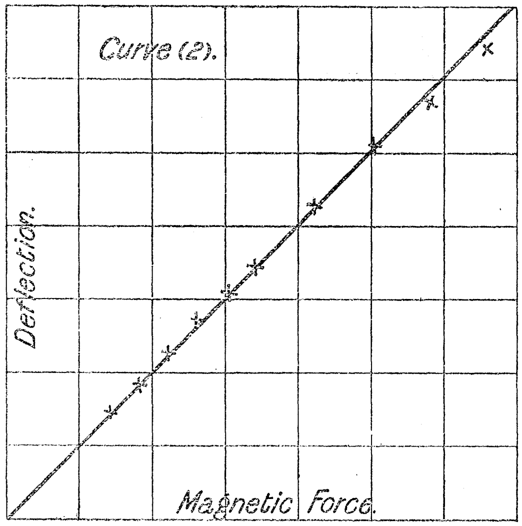
we see that the magnetic force acting on the needle is proportional to the length of the arc traversed by the discharge.

A series of observations were made and it was found that the deflection due to the detector was approximately proportional to the magnetic force acting on the needle, provided the magnetic force was well below the value required to completely demagnetize the steel.

Curve (2) represents the relation between the deflection of the magnetometer and the magnetic force acting on the needle. The curve is nearly a straight line except near the top part of the curve.

To determine the damping of the oscillations a discharge was passed in one direction and the deflection noted. The detector was removed, magnetized and replaced. The direction of the discharge was reversed and the arm of the circle moved until the deflection was the same as before. When this is the case the ratio of the maximum values of the first and second half oscillation is given by the ratios of the arcs traversed by the discharge.

In this way the rate of decay of oscillations in ordinary discharge circuits was examined. With short air breaks and copper wires for connection, it was found that the damping was hardly appreciable. As the length of the spark gap was increased, the absorption of energy in the air-break caused the oscillations to damp rapidly.

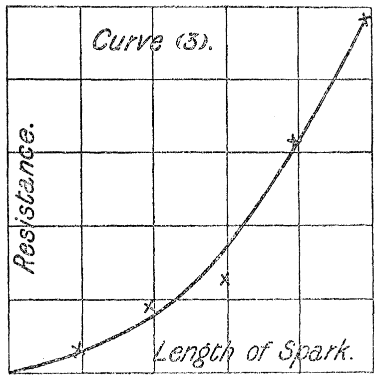


If the copper wires of the discharge circuit were replaced by iron wires, there was in all cases a very rapid decay of the oscillations, whatever the length of the air-break. If an iron cylinder were placed in a solenoid, the absorption of energy by the cylinder caused a rapid damping, while a copper cylinder of the same diameter had no appreciable effect.

Experiments on Damping of Oscillations. Discharge circuit rectangular, 184 centims. by 90 centims.; Self inductance of circuit, L = 7,400; Capacity, C = 2,000 electrostatic units; Frequency, 1·25 millions per second.

Length of spark gap.	Ratio of amplitudes of two first half oscillations.	Resistance.
centim.		ohms.
·06	·98	—
·12	·97	1·1
·24	·93	2·6
·37	·9	3·7
·49	·79	8·4
·61	·7	12·4

In the third column the apparent resistance, corresponding to the absorption of energy in the conducting wires and spark gap is tabulated. The calculated value of the resistance of the wires of the discharge circuit was ·4 ohm, so that the remainder of the resistance is due to the great absorption of energy in the air break.



The above curve represents the relation between the length of the spark and the apparent resistance that the spark offers to the discharge. The ohmic resistance of the air break is probably very variable, depending on the intensity of the charge at any instant, but the absorption of energy is quite definite and may be expressed in terms of the non-inductive resistance which, when placed in the circuit, would absorb the same amount of energy.

It will be observed that the damping of the oscillations increases rapidly with the length of the spark, and that the resistance of the air break increases very rapidly with its length.

It was also found that the damping depended on the capacity when the inductance and spark length were kept constant. The damping and also the resistance of the spark were found to increase with increase of capacity. For example with an air break of ·32 centim. the damping and resistance are given below.

Capacity.	Ratio of Oscillations.	Resistance.
1000	·94	2·2
2000	·9	2·6
4000	·81	3·8

When the capacity of the circuit was small the damping was found to be very small. If iron wires were put into the place of the copper wires in the discharge circuit, the damping was found to be great for all capacities investigated.

When the capacity of the circuit was only 130 electrostatic units, and inductance 2400, no appreciable damping was found for an air break ·5 centim.: When the copper wire was replaced by an iron one of the same dimensions, the second half-oscillation was only ·6 of the amplitude of the first.

Resistance of Iron Wires for High Frequency Discharges.

The rapid decay of the oscillations when iron wires formed the discharge circuit has been already noted. This has been observed by TROWBRIDGE ('Phil. Mag.,' December, 1891), who found, by photographing the spark, that there was evidence of much fewer oscillations when iron wires were used instead of copper.

For very rapid oscillations the resistance R′ is given by $R' = \sqrt{\frac{1}{2}p\mu l R}$ (see Lord RAYLEIGH, "On the Self-Induction and Resistance of Straight Conductors," 'Phil. Mag.,' 1886), where R is the resistance of the wire for steady currents, l the length of the wire, μ the permeability, and $p = 2\pi n$, where n is the number of oscillations per second.

Since the expression involves μ, we should expect the resistance to be much greater for iron wires than for wires of the same conductivity, but non-magnetic.

To determine the resistance of iron wires a very simple method was used. The fall of deflection, due to the detector needle, arranged as in (fig. 2) was noted. The iron wire was then removed and a copper one of the same diameter substituted. Since the inductance of the circuit was practically unchanged, if the damping in the two circuits are equal, the resistances should be the same. A short piece of high-resistance platinoid wire was introduced into the circuit of the copper wire, and the length adjusted until the deflection was the same as in the first case. When this is so, the resistance of the platinoid wire, together with the resistance of the copper wire, is equal to the resistance of the iron wires for the frequency employed.

The resistance of the copper wires was calculated for the frequency used, but was, in general, small compared with the resistance introduced.

The resistance of the platinoid wire was also calculated, but was found to be practically the same as for steady currents. The length of wire placed in the current was 265 centims. ; spark length was ·25 centim.

Kind of wire.	Diameter.	R.	R_1.	R_1/R.
	centims.			
Soft iron	·025	6·1	11·8	1·9
,, ...	·047	2·62	12·8	4·9
,, ...	·094	·57	9·2	16
,, ...	·295	·051	4·2	72
Pianoforte steel wire . . .	·062	1·51	11·3	6·5
Nickel wire	·062	·66	3·7	5 9

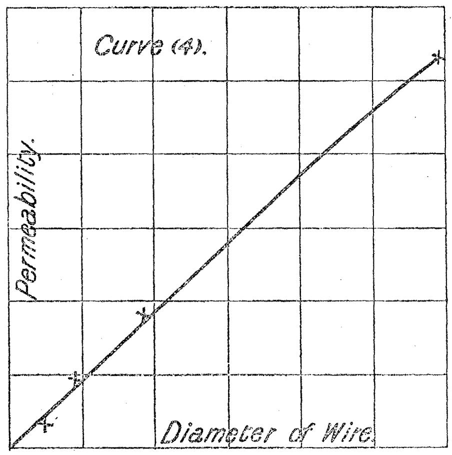
In the above Table, R is the resistance for steady currents, R_1 the resistance for a frequency 1·6 millims. per second. The last column gives the ratio R_1/R for the different wires. In the case of the wire ·295 centim. in diameter, the resistance is 72 times the resistance for steady currents, while in the case of the wire of ·025 centim. diameter, it is only 1·2 times.

If the value of μ, the permeability of the specimens of soft iron, be calculated from the formula

$$R' = \sqrt{\tfrac{1}{2}p\mu l}R,$$

it will be found that the value varies with the diameter of the wire.

Diameter.	μ.
centims.	
·025	3·5
·042	9·4
·094	18
·295	53



The above curve (Curve 4) shows the relation between the diameter of the wire and the permeability for the discharge. It will be seen that μ varies approximately as the radius of the wire.

The very small value of the permeability for fine wires is to be expected when we consider the very large currents that pass through the wire, and the consequent large value of the magnetic force that acts at the surface of the iron.

The maximum current of discharge, assuming the damping to be small, is given by

$\gamma = pCV_0$, where C is capacity and V_0 the potential to which condenser is charged.

In the above experiments the air-break was about $\frac{1}{10}$ inch, and the difference of potential about 10,000 volts, and since $p = 5 \cdot 10^6$, $C = 4000$, the maximum current was about 222 amperes.

The magnetic force at the surface of the wire of radius r, which conveys the current, is given by

$$H = \frac{2\gamma}{r} = 3552 \text{ C.G.S. units, if } r = \cdot 0125 \text{ centim.}$$

If we assume the value of B to be about 14,000, we see that the permeability for the extreme surface layer would be about 4. The value of the magnetizing force diminishes from the surface inwards, so that the mean permeability of the iron to the discharge should be greater than the value at the surface. These considerations show that the permeability of iron to these discharges is by no means constant, but depends on the diameter of the wire and the intensity of the discharge.

The resistance of iron wires was found to vary with the length of the spark. Short air-breaks gave higher values of the resistance than long ones. When the length of the spark was so adjusted that the maximum current was approximately constant for different periods, the resistance was found to vary as the square root of the frequency, as we should expect from theory.

Several specimens of pianoforte steel wire were examined to see whether the larger waste of energy due to hysteresis in steel materially affected the value of the resistance, but the increase of resistance was not so great as for soft iron wires of the same diameter, although the loss due to hysteresis in steel is much greater than in soft iron for slow cycles.

Absorption of Energy by Metal Cylinders.

This subject has been treated mathematically and experimentally by J. J. Thomson ('Recent Researches,' p. 321–326). It is there shown by observing the electrodeless discharge that a cylinder of iron placed in a solenoid absorbs considerably more energy than a copper one of the same dimensions. The method adopted here admitted of quantitative as well as qualitative results.

An ordinary Leyden jar was discharged through a solenoid of about thirty turns and 14 centims. long. The metal cylinder was then placed in the solenoid and the damping of the oscillations observed. The cylinder was then removed and a non-inductive resistance added until the damping was the same as when the metal cylinder was in the solenoid. The absorption of energy in the cylinder was then equal to the absorption of energy in the added resistance, whose value was known. In the above we have taken no account of the change of inductance of the circuit due to the metal cylinder being placed in the solenoid. The change is small, and could be made negligible by making the inductance of the solenoid small compared with the rest of the circuit.

(1.) A test-tube was filled with finely laminated iron wire, ·008 inch in diameter. The test-tube was filled with paraffin oil, to insure insulation from eddy currents.

The absorption of energy in this case corresponded to an added resistance of 10·25 ohms to the circuit.

(2.) A test-tube filled with steel filings and insulated as in (1). Increase of resistance, 9 ohms.

(3.) A thin soft iron cylinder, 1·9 centims. in diameter. Increase of resistance, 3·9 ohms.

(4.) Solid iron rod. Increase of resistance, 3·3 ohms.

(5.) A copper cylinder, a test-tube filled with a copper sulphate solution, and a platinum cylinder showed no appreciable absorption of energy.

(6.) A carbon rod absorbed a large amount of energy. Increase of resistance, 3·3 ohms.

TABLE of Absorption of Energy of various Conductors; Absorption of Energy expressed in Terms of the Increased Resistance of the Discharge Circuit.

Substance.	Increase of resistance.
Laminated soft iron wires . . .	10·25 ohms
Solid soft iron cylinder	3·5 ,,
Hollow iron cylinder	3·9 ,,
Carbon cylinder	3·3 ,,
Copper, platinum, zinc cylinders .	not appreciable
Steel filings	9 ohms

The frequency of the oscillations in the above experiment was two million per second.

If the experimental value obtained for the increase of resistance due to the solid iron cylinder be compared with the theoretical value ('Recent Researches,' p. 323), the value of the permeability will be found to be 172, which accounts for the much greater absorption by an iron cylinder than a copper one.

From the peculiar deadened sound of the spark, it could always be told when much energy was being absorbed in the discharge circuit. With copper wires for the discharge circuit, the spark was sharp and bright; when iron wires were substituted, the spark was weak; when an iron cylinder was put in the place of a copper one, the spark was neither so bright, nor so sharp in sound.

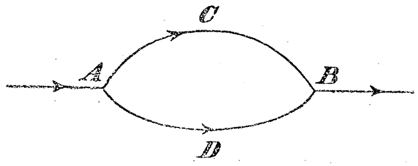
Determination of the Period of a Discharge Circuit.

It is often a difficult matter to obtain even an approximation of the period of oscillation of a discharge circuit when the capacity of the condenser and the self-inductance of the circuit cannot be directly calculated.

The following simple method was found to work very accurately in practice, and could be used for a fairly wide range of frequencies.

Let ACB, ADB (fig. 5) be two branches of a discharge circuit in parallel, R and L the resistance and inductance of the branch ACB, S and N the resistance and inductance of branch ADB.

Fig. 5.



Let M be the coefficient of mutual inductance between the two branches.

Let x and y be currents in branches ACB, ADB respectively.

It is shown ('Recent Researches,' p. 513) that for a rapidly-alternating current of frequency n, where $p = 2\pi n$, that

$$x = \left\{\frac{S^2 + (N - M)^2 p^2}{(L + N - 2M)^2 + (R + S)^2}\right\}^{\frac{1}{2}} \cos(pt + \epsilon) = A \cos(pt + \epsilon), \text{ say}$$

$$y = \left\{\frac{R^2 + (L - M)^2 p^2}{(L + N - 2M)^2 + (R + S)^2}\right\}^{\frac{1}{2}} \cos(pt + \epsilon') = B \cos(pt + \epsilon'),$$

$$\tan \epsilon = \frac{p\{R(N - M) - S(L - M)\}}{S(R + S) + (L + N - 2M)(N - M)p^2},$$

$$\tan \epsilon' = - \frac{p\{R(N - M) - S(L - M)\}}{R(R + S) + (L + N - 2M)(L - M)p^2}.$$

A and B are the maximum currents in the two branches ACB, ADB respectively, and

$$\frac{A}{B} = \sqrt{\frac{S^2 + (N - M)^2 p^2}{R^2 + (L - M)^2 p^2}}.$$

If the circuits be so adjusted that A = B

$$R^2 + (L - M)^2 p^2 = S^2 + (N - M)^2 p^2,$$

and

$$p^2 = \frac{R^2 - S^2}{N^2 - L^2 - 2M(N - L)}.$$

The value of the impedance $\sqrt{R^2 + p^2 L^2}$ is nearly independent of R for rapid frequencies in ordinary copper wire circuits

Suppose

$$n = 10^6, \qquad p = 2\pi . 10^6, \qquad \text{and} \qquad L = 10^4,$$

then

$$p^2 L^2 = 4\pi^2 . 10^{20}.$$

If the value of R for the particular period was 2 ohms say, then

$$\frac{R^2}{p^2 L^2} = \frac{1}{100\pi^2}$$

a very small quantity.

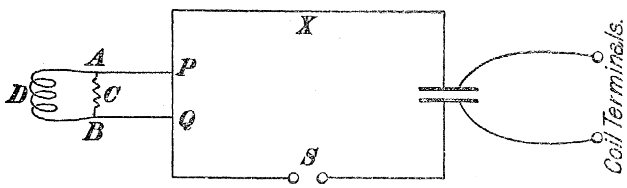
We therefore see that, under ordinary circumstances, the resistances may be neglected in comparison with the inductances.

If the branch ACB of the divided circuit consist of a high resistance of short length, and consequently small inductance, and the other branch of an inductance N, and the maximum values of the currents in the two branches are equal, then $p^2 = \frac{R^2}{N^2 - L^2}$, since S^2 may be neglected in comparison with R^2, and supposing the value of M to be small compared with N.

If the inductances are unchanged, $N^2 - L^2$ is practically a constant for all periods, and p is therefore proportional to R.

In practice, one branch of the divided circuit consisted of a standard inductance, N, and the other branch of an electrolytic resistance, R. The equality of currents in the two circuits was obtained by altering the value of R until the effect on the detector needles was the same for both circuits. Since the effect on the needle was the same in both circuits, the maximum values of the current are the same, since each branch is traversed by an oscillation of the same period.

Fig. 4.



It was not found necessary to place the divided circuit in series with the discharge circuit, but the arrangement was more satisfactory when it was shunted off a portion, PQ, of the discharge circuit XPQS (fig. 6), whose period is to be determined. The addition of the shunt circuit had no appreciable effect on the period of the oscillation for the equivalent inductance of the two branches QP, QBAP, was slightly less than that of QP, and the length of QP was generally not a tenth part of the whole discharge circuit.

ACB, ADB are the branches of the divided circuit. In C was placed a resistance consisting of zinc sulphate with zinc electrodes. The amount of resistance in the current could be varied by altering the length of electrolyte through which the current passed.

In D was placed a standard inductance consisting of six turns of insulated wire wound on a bobbin 10 centims. in diameter. The self-inductance of this could be accurately determined by calculation, and was very approximately the same for steady as for rapidly changing fields.

If the inductance L of the resistance branch was small compared with N, the value of p is given by $p = R/N$.

Even if $L = \frac{N}{10}$ the correction would only be 1 per cent. The resistance R of the zinc sulphate solution was determined for steady currents, and it can be shown that the change of resistance due to the concentration of the currents on the surface is quite inappreciable for the periods investigated on account of the high specific resistance of the solution.

This can experimentally be shown as follows: a tube containing the solution to be tested is placed inside a solenoid of a few turns, and a detector needle placed in the solution. After the passage of a discharge it will be found that the effect on the needle is the same as when the solution is removed, showing that there is no screening action on the needle due to the solution. Since the law of decrease of magnetic force from the surface inwards is the same as for the decrease of amplitude of a current through the conductor, it follows that the amplitude of the current at the centre of the solution was the same as at the surface, and that there was no alteration of the resistance of the electrolyte due to concentration of the current on the surface.

An air condenser of calculable capacity C was discharged through a circuit whose inductance L for rapid frequencies could be very approximately determined.

The value of $p = \frac{1}{\sqrt{LC}}$ obtained from theory was found to agree to within 3 per cent. of the experimentally-determined value, and, from the difficulty of accurately calculating the inductance, it is probable that the experimental determination is nearer the true value.

From the close agreement of theory and experiment, we have indirectly proved that the resistance of an electrolyte like zinc sulphate is the same for high frequencies as for low.

As an example of the determination of the period of oscillation, the value of N, the standard inductance, was 6500 units. When the current was the same in both circuits, the value of R was 168 ohms.

Therefore

$$p = \frac{R}{N} = \frac{168 \cdot 10^9}{6500} = 2 \cdot 6 \ 10^7.$$

The frequency $n = p/2\pi = 4 \cdot 1 \ 10^6$.

The value of the capacity and the inductance for rapid frequencies of the discharge circuit could also be determined.

If a Leyden jar of unknown capacity C be replaced by an air condenser of known capacity C′, the value of L remaining unaltered, and the value of the resistance necessary for equality of currents in the two circuits determined as before, then if p' and R′ be the new values of p and R,

$$p = \frac{1}{\sqrt{LC}} \quad \text{and} \quad p' = \frac{1}{\sqrt{LC'}},$$

$$R = pN \quad \text{and} \quad R' = p'N.$$

Therefore

$$\frac{C}{C'} = \frac{p'^2}{p^2} = \frac{R'^2}{R^2}.$$

Since R and R′ have both been determined, C is known in terms of the standard capacity C′.

Similarly, the value of L, the self-inductance for rapid frequencies, may also be found. If an additional standard inductance L′ be introduced into the circuit, the value of the capacity remaining unaltered,

$$p = \frac{1}{\sqrt{LC}} \quad \text{and} \quad p' = \frac{1}{\sqrt{(L + L')C}}.$$

Therefore

$$\frac{L + L'}{L} = \frac{p^2}{p'^2} = \frac{R^2}{R'^2}.$$

The value of L for rapid frequencies is thus determined in terms of L′, a known inductance.

In the experiments on the determination of periods, a detector consisting of twenty or more fine insulated steel wires, about 1 centim. long, was used, with one or two turns of wire round it through which the oscillatory current passed. This small detector coil was fixed before a magnetometer, and was so arranged that it could be switched either into the resistance or inductance branch of the divided circuit.

The inductance of the detector coil was too small to appreciably alter the distribution of the current in either circuit. The equality of currents in the two circuits could thus be readily compared.

The "longitudinal" detector may also be used for rough determinations; but it is not so sensitive to slight changes of current as the solenoidal detector of fine wire.

It was found that the inductance of a current when the wire was of iron was nearly the same as when replaced by copper of the same diameter. It was difficult to determine the variation of inductance accurately in this case, in consequence of the oscillations being rapidly damped when iron wires were used.

Since the capacities of condensers for very rapid alternations may be determined, it was interesting to observe whether the values of the specific inductive capacity of glass was the same for slow as for very rapidly varying fields. Some observers had found that glass had a much lower specific inductive capacity for rapid oscillations than for slow, while others again found values about the same in the two cases. The value of the specific inductive capacity found for plate-glass was about 6·5 for periods of about 3 million per second. This is considerably higher than the value obtained for plate-glass by J. J. THOMSON and BLONDLOT, who found values of 2·7 and 2·3 respectively for periods of about 20 million per second.

The value of the specific inductive capacity of ebonite, tested by the same method, was found to be about the same as for slow alternations.

These experiments were performed in the Cavendish Laboratory, Cambridge.